

Resource Allocation with Subcarrier Pairing in OFDMA Two-Way Relay Networks

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Abstract—This study considers an orthogonal frequency-division multiple-access (OFDMA)-based multi-user two-way relay network where multiple mobile stations (MSs) communicate with a common base station (BS) via multiple relay stations (RSs). We study the joint optimization problem of subcarrier-pairing based relay-power allocation, relay selection, and subcarrier assignment. The problem is formulated as a mixed integer programming problem. By using the dual method, we propose an efficient algorithm to solve the problem in an *asymptotically* optimal manner. Simulation results show that the proposed method can improve system performance significantly over the conventional methods.

Index Terms—Two-way relaying, subcarrier pairing, resource allocation, orthogonal frequency-division multiple-access.

I. INTRODUCTION

An important property of orthogonal frequency-division multiplexing (OFDM)-based relaying is that the frequency diversity can be exploited by *subcarrier pairing*, which matches the incoming and outgoing subcarriers at the relay based on channel dynamics and hence improves system performance. In multi-user environments with orthogonal frequency-division multiple-access (OFDMA), subcarriers should not only be carefully paired at the relay but also be assigned adaptively for different users. If with multiple relays, it further complicates the problem because relay selection tightly couples with subcarrier pairing and assignment. Thus, subcarrier-pairing based resource allocation in multi-user multi-relay OFDMA networks is highly challenging.

Subcarrier-pairing based resource allocation has been originally investigated for single-user single-relay one-way communications (e.g., [1], [2]). In particular, it is proved in [1] that the *ordered pairing* is optimal for amplify-and-forward (AF) protocol. Authors in [3] investigated *separated* power allocation and subcarrier pairing in two-way communication using single relay, where the power allocation is first employed by water-filling and then subcarriers are paired at the relay by a heuristic method. In [4], the subcarrier-pairing based joint optimization of power allocation, relay selection and subcarrier assignment for single-user multi-relay systems was studied. The subcarrier-pairing based joint optimization of power allocation and subcarrier-user assignment for multi-user single-relay scenario was studied in [5]. In [6], the authors

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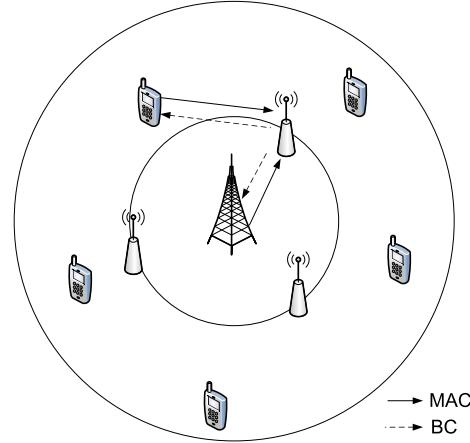


Fig. 1. System model.

studied relay-assisted bidirectional OFDMA cellular networks, wherein the subcarrier-pairing based joint optimization of bidirectional transmission mode selection, relay selection, and subcarrier assignment was investigated by a graph approach. Authors in [7] investigated the jointly optimal channel and relay assignment for multi-user multi-relay two-way relay networks. These works [6], [7], however, did not consider power allocation.

In this work, we consider an OFDMA two-way relay network with a common base station (BS), multiple mobile stations (MSs) and multiple relay stations (RSs). The downlink and uplink traffic for each MS is multiplexed through analog network coding at the RSs. We formulate a joint optimization problem of subcarrier-pairing based relay-power allocation, relay selection, and subcarrier assignment. The problem is a mixed integer programming problem and we solve it efficiently in dual domain with polynomial complexity.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a single-cell OFDMA two-way relay network, as shown in Fig. 1, with one BS, multiple MSs and RSs. All the MSs are assumed to be cell-edge users so that both the downlink and uplink traffic of each user needs to be relayed through one or more RSs. This assumption is commonly used for cellular relay networks in the literature (e.g., [5], [8], [9]). Each RS operates in a half-duplex mode and relays the bi-directional traffic using AF protocol, known as analog network coding. In specific, the AF two-way relay protocol takes place in two phases [10]. In the first phase, also known as multiple-access (MAC) phase, all the MSs and the BS concurrently transmit signals while all the RSs listen. In

the second phase, known as broadcast (BC) phase, the RSs amplify the received signals and then forward them to the MSs and the BS. To avoid multi-user interference, each MS and RS operate in non-overlapping subcarriers in the first and second phases, respectively. The downlink-uplink interference within each user is eliminated by self-interference cancelation. Furthermore, the channel is assumed to be slowly time-varying and all the channel state information can be perfectly estimated and known at the BS for centralized processing.

Let $\mathcal{N} = \{1, 2, \dots, N\}$ denote the set of subcarriers, $\mathcal{K} = \{1, 2, \dots, K\}$ denote the set of RSs, and $\mathcal{U} = \{1, 2, \dots, M\}$ denote the set of MSs. The channel coefficients from BS b and MS u to RS k on subcarrier i in the MAC phase are denoted as $h_{b,k,i}$ and $f_{u,k,i}$, respectively, $\forall u \in \mathcal{U}, k \in \mathcal{K}, i \in \mathcal{N}$. In the BC phase, the channel coefficients from RS k to BS b and MS u on subcarrier j are denoted as $h_{k,b,j}$ and $f_{k,u,j}$, respectively, $\forall u \in \mathcal{U}, k \in \mathcal{K}, j \in \mathcal{N}$. Here channel reciprocity is used, which is valid in TDD (time-division duplex) mode. Along with the paths, we further denote $p_{b,k,i}$ and $p_{u,k,i}$ as the transmitted power of BS b and MS u respectively, and $p_{k,u,j}$ as the transmitted power of RS k . Then, the sum-rate of uplink-downlink transmission of MS u over subcarrier pair (i, j) with the assistance of RS k can be expressed as [3], [10]

$$R_{u,k,i,j} = \frac{1}{2} \log_2 \left(1 + \frac{p_{u,k,i} |f_{u,k,i}|^2 p_{k,u,j} |h_{k,b,j}|^2}{p_{k,u,j} |h_{k,b,j}|^2 + m_{u,k,i}} \right) + \frac{1}{2} \log_2 \left(1 + \frac{p_{b,k,i} |h_{b,k,i}|^2 p_{k,u,j} |f_{k,u,j}|^2}{p_{k,u,j} |f_{k,u,j}|^2 + m_{u,k,i}} \right), \quad (1)$$

in which $m_{u,k,i} = 1 + p_{b,k,i} |h_{b,k,i}|^2 + p_{u,k,i} |f_{u,k,i}|^2$. It can be proved that the sum-rate $R_{u,k,i,j}$ is concave in the relay power $p_{k,u,j}$.

We then introduce a set of binary variables $\rho_{u,k,i,j} \in \{0, 1\}$ for all u, k, i, j . When $\rho_{u,k,i,j} = 1$, it means that subcarrier i in the MAC phase is paired with subcarrier j in the BC phase and they are used by RS k to relay the signals of MS u . Otherwise, we have $\rho_{u,k,i,j} = 0$. These binary variables must satisfy the following constraints, due to the exclusive subcarrier assignment,

$$\sum_{u=1}^M \sum_{k=1}^K \sum_{j=1}^N \rho_{u,k,i,j} \leq 1, \quad \forall i, \quad (2)$$

$$\sum_{u=1}^M \sum_{k=1}^K \sum_{i=1}^N \rho_{u,k,i,j} \leq 1, \quad \forall j. \quad (3)$$

For simplicity, we study relay-power allocation and let the transmit power of the BS and MSs be fixed. Each RS is subject to its own peak power constraint. This can be expressed as:

$$\sum_{u=1}^M \sum_{j=1}^N p_{k,u,j} \leq P_k, \quad \forall k, \quad (4)$$

where P_k is the peak power constraint of RS k .

Our objective is to maximize the system total weighted throughput by jointly optimizing the assignment variables $\{\rho_{u,k,i,j}\}$ and the relay power variables $\{p_{k,u,j}\}$. Mathemati-

cally, this can be formulated as:

$$\begin{aligned} \max_{\{\mathbf{p}, \boldsymbol{\rho}\}} & \sum_{u=1}^M w_u \sum_{k=1}^K \sum_{i=1}^N \sum_{j=1}^N \rho_{u,k,i,j} R_{u,k,i,j} (p_{k,u,j}) \\ \text{s.t.} & (2), (3), (4), \end{aligned} \quad (5)$$

where w_u is the weight that represents the priority of MS u , $\mathbf{p} \in \mathbb{R}_+^{K \times M \times N}$ and $\boldsymbol{\rho} \in \{0, 1\}^{M \times K \times N \times N}$ are matrices with entries $p_{k,u,j}$ and $\rho_{u,k,i,j}$, respectively.

III. DUAL BASED ALGORITHM

We first define \mathcal{T} as the set of all possible $\boldsymbol{\rho}$ satisfying (2) and (3), \mathcal{P} as the set of all possible power allocations \mathbf{p} for the given $\boldsymbol{\rho}$ that satisfy $p_{k,u,j} \geq 0$ for $\rho_{u,k,i,j} = 1$ and $p_{k,u,j} = 0$ for $\rho_{u,k,i,j} = 0$. Denote $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_K) \succeq 0$ as the dual variables associated with the peak power constraints of the RSs. Then the dual function of the problem in (5) can be defined as

$$g(\boldsymbol{\lambda}) \triangleq \max_{\substack{\mathbf{p} \in \mathcal{P}(\boldsymbol{\rho}) \\ \boldsymbol{\rho} \in \mathcal{T}}} L(\mathbf{p}, \boldsymbol{\rho}, \boldsymbol{\lambda}), \quad (6)$$

where the Lagrangian is

$$L(\mathbf{p}, \boldsymbol{\rho}, \boldsymbol{\lambda}) = \sum_{u=1}^M w_u \sum_{k=1}^K \sum_{i=1}^N \sum_{j=1}^N R_{u,k,i,j} (p_{k,u,j}) + \sum_{k=1}^K \lambda_k \left(P_k - \sum_{u=1}^M \sum_{j=1}^N p_{k,u,j} \right). \quad (7)$$

Computing the dual function $g(\boldsymbol{\lambda})$ requires us to determine the optimal $(\mathbf{p}, \boldsymbol{\rho})$ at the given dual vector $\boldsymbol{\lambda}$. In the following we present the derivations in detail.

A. Optimizing the Primal Variables $(\mathbf{p}, \boldsymbol{\rho})$ for Given $\boldsymbol{\lambda}$

We first find the optimal power variables \mathbf{p} by fixing the binary assignment variables $\boldsymbol{\rho}$. Then we search the optimal $\boldsymbol{\rho}$ by eliminating \mathbf{p} in the objective function. Such a method has been commonly used in the literature (e.g., [2], [4], [5], [11]).

Let us rewrite $L(\mathbf{p}, \boldsymbol{\rho}, \boldsymbol{\lambda})$ as

$$L(\mathbf{p}, \boldsymbol{\rho}, \boldsymbol{\lambda}) = \sum_{k=1}^K \sum_{u=1}^M \sum_{j=1}^N L_{u,k,j} (p_{k,u,j}) + \sum_{k=1}^K \lambda_k P_k, \quad (8)$$

where

$$L_{u,k,j} (p_{u,k,j}) = w_u \sum_{i=1}^N R_{u,k,i,j} (p_{k,u,j}) - \lambda_k p_{k,u,j}. \quad (9)$$

Suppose $\rho_{u,k,i,j} = 1$ for a certain (u, k, i, j) . It is easy to verify that $L_{u,k,j} (p_{u,k,j})$ is concave in $p_{k,u,j}$ and thus the optimal $p_{k,u,j}^* (\lambda_k)$ can be obtained by applying the Karush-Kuhn-Tucker (KKT) conditions. More specifically, $p_{k,u,j}^* (\lambda_k)$ is the non-negative real root of the following quartic function

$$ap_{k,u,j}^4 + bp_{k,u,j}^3 + cp_{k,u,j}^2 + dp_{k,u,j} + e = 0, \quad (10)$$

where a, b, c, d, e are coefficients determined by the dual variables, MSs' weights, and channel gains as defined at the top of the next page.

$$\begin{aligned}
a &= 2 \ln 2 \lambda_k |h_{b,k,j}|^4 |f_{u,k,j}|^4 / m_{u,k,i}, \\
b &= 4 \ln 2 \lambda_k |h_{b,k,j}|^2 |f_{u,k,j}|^2 (|f_{u,k,j}|^2 + |h_{b,k,j}|^2), \\
c &= 2m_{u,k,i} \ln 2 \lambda_k (|h_{b,k,j}|^4 + |f_{u,k,j}|^4 + 4|h_{b,k,j}|^2 |f_{u,k,j}|^2) \\
&\quad - w_u |h_{b,k,j}|^2 |f_{u,k,j}|^2 (p_{u,k,i} |f_{u,k,i}|^2 |f_{u,k,j}|^2 + p_{b,k,i} |h_{b,k,i}|^2 |h_{b,k,j}|^2), \\
d &= 4m_{u,k,i}^2 \ln 2 \lambda_k (|f_{u,k,j}|^2 + |h_{b,k,j}|^2) - 2w_u m_{u,k,i} |h_{b,k,j}|^2 |f_{u,k,j}|^2 ((p_{u,k,i} |f_{u,k,i}|^2 + p_{b,k,i} |h_{b,k,i}|^2), \\
e &= 2m_{u,k,i}^3 \ln 2 \lambda_k - w_u m_{u,k,i}^2 (p_{u,k,i} |f_{u,k,i}|^2 |h_{b,k,j}|^2 + p_{b,k,i} |h_{b,k,i}|^2 |f_{u,k,j}|^2).
\end{aligned}$$

Substituting the optimal power allocations $\mathbf{p}^*(\boldsymbol{\lambda})$ into (6) to eliminate the power variables, the dual function can be rewritten as

$$g(\boldsymbol{\lambda}) = \max_{\boldsymbol{\rho} \in \mathcal{T}} \sum_{u=1}^M \sum_{k=1}^K \sum_{i=1}^N \sum_{j=1}^N \rho_{u,k,i,j} X_{u,k,i,j} + \sum_{k=1}^K \lambda_k P_k,$$

where

$$X_{u,k,i,j} = w_u R_{u,k,i,j} (p_{k,u,j}^*(\lambda_k)) - \lambda_k p_{k,u,j}^*(\lambda_k). \quad (11)$$

Now we are ready to find the optimal $\boldsymbol{\rho}$. In the following, we show that $X_{u,k,i,j}$ defined in (11) plays an important role in user and relay selection for occupying a subcarrier pair (i, j) .

Noting the constraints (2) and (3), we conclude that there is at most one non-zero element for a given subcarrier pair (i, j) . This suggests that at most one MS and one RS can occupy the subcarrier pair (i, j) . Based on the observation, we define

$$\mathcal{X}_{i,j} = \max_{k \in \mathcal{K}, u \in \mathcal{U}} X_{u,k,i,j}, \quad (12)$$

$$(u^*, k^*)_{i,j} = \arg \max_{k \in \mathcal{K}, u \in \mathcal{U}} X_{u,k,i,j}. \quad (13)$$

Then the dual function can be further written as

$$g(\boldsymbol{\lambda}) = \max_{\boldsymbol{\rho} \in \mathcal{T}} \sum_{i=1}^N \sum_{j=1}^N \rho_{u^*, k^*, i, j} \mathcal{X}_{i,j} + \sum_{k=1}^K \lambda_k P_k. \quad (14)$$

From (14) it can be seen that if subcarrier i in the MAC phase is paired with subcarrier j in the BC phase, then the pair should be used by MS u^* with the help of RS k^* , i.e., the MS and RS with the maximum $X_{u,k,i,j}$ as defined in (11). This can be interpreted from an economic perspective. Suppose each dual variable λ_k represents the power price of RS k . Then $X_{u,k,i,j}$ can be regarded as the profit of letting MS u transmitting over the subcarrier pair (i, j) with the help of RS k , where the profit is defined as the throughput revenue $w_u R_{u,k,i,j}$ minus the power cost $\lambda_k p_{k,u,j}^*$. Clearly, to maximize the system total profit, each potential subcarrier pair (i, j) should be assigned to the MS and RS that can generate the maximum sub-profit.

The remaining problem is then to identify the optimal subcarrier pairings $\rho_{u^*, k^*, i, j}$. This is a standard *two-dimensional assignment problem*. The classical Hungarian method can be applied to find the optimal $\boldsymbol{\rho}^*(\boldsymbol{\lambda})$ in polynomial time.

B. Optimizing the Dual Vector $\boldsymbol{\lambda}$

After computing $g(\boldsymbol{\lambda})$, we now solve the standard dual optimization problem which is

$$\begin{aligned}
\min_{\boldsymbol{\lambda}} \quad & g(\boldsymbol{\lambda}) \\
\text{s.t.} \quad & \boldsymbol{\lambda} \succeq 0.
\end{aligned} \quad (15)$$

Since a dual function is always convex, subgradient-based methods can be used to minimize $g(\boldsymbol{\lambda})$ with global convergence with the fact that

$$\Delta \lambda_k = P_k - \sum_{u=1}^M \sum_{j=1}^N p_{k,u,j}^*(\lambda_k) \quad (16)$$

is the subgradient at $\lambda_k, \forall k$. In specific, denote $\Delta \boldsymbol{\lambda}^{(l)} = (\Delta \lambda_1^{(l)}, \Delta \lambda_2^{(l)}, \dots, \Delta \lambda_K^{(l)})$, then we can update the dual variables as $\boldsymbol{\lambda}^{(l+1)} = \boldsymbol{\lambda}^{(l)} + \omega^{(l)} \Delta \boldsymbol{\lambda}^{(l)}$. Here, $\omega^{(l)}$ is the diminishing step size at the l th iteration to guarantee the convergence of the subgradient method.

C. Refinement of Power Allocation

Having the dual point $\boldsymbol{\lambda}^*$, we now need to determine the optimal solution to the primal problem (5). Due to the non-zero duality gap, the optimal $\boldsymbol{\rho}^*(\boldsymbol{\lambda}^*)$ and $\mathbf{p}^*(\boldsymbol{\lambda}^*)$ may not satisfy all the constraints (2), (3), and (4) in the original problem. To overcome this problem, we first determine the optimal assignment $\boldsymbol{\rho}^*$ in dual domain, and then make a refinement of the power allocation to meet the power constraints in the primal problem. More specifically, denote $\mathcal{A}_{u,k}$ as the set of active subcarrier pairs assigned to MS u and RS k obtained from the dual problem. The problem can be written as:

$$\max_{\boldsymbol{p}} \sum_{u=1}^M w_u \sum_{k=1}^K \sum_{(i,j) \in \mathcal{A}_{u,k}} R_{u,k,i,j} (p_{k,u,j}) \quad (17)$$

$$\text{s.t.} \quad \sum_{u=1}^M \sum_{(i,j) \in \mathcal{A}_{u,k}} p_{k,u,j} \leq P_k, \quad \forall k. \quad (18)$$

Clearly, this is a convex problem. By applying KKT conditions, we can obtain the optimal $p_{k,u,j}^*$ which has the same expression as that in the dual domain.

Finally we summarize the overall procedure of the proposed dual-based solution in Algorithm 1. This algorithm is asymptotically optimal when N is sufficiently large [12].

Algorithm 1 Proposed algorithm for problem (5)

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1: initialize  $\lambda^{(0)}$  as a random non-negative vector,  $l = 0$ .
2: repeat
3:   Compute  $X_{u,k,i,j}$  using (11) for all  $(u, k, i, j)$  with  $p_{k,u,j}^*$  being the non-negative real root of (10).
4:   Obtain  $\mathcal{X}_{i,j}$  and  $(u^*, k^*)$  using (12) and (13) respectively for all  $(i, j)$ , then obtain optimal  $\rho^*(\lambda^{(l)})$  by solving (14).
5:   Update  $\lambda^{(l)}$  using the subgradients  $\Delta\lambda^{(l)}$  in (16); Let  $l \leftarrow l + 1$ .
6: until  $\lambda$  converges.
7: Set the final  $\rho$  as  $\rho^*$  obtained in the dual domain and refine the power parameter  $p^*$  by solving (17) at the given  $\rho$ .

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D. Discussion on Complexity and Proportional Fairness

The complexity of updating the dual variables λ is $\mathcal{O}(K^q)$ (e.g., if the ellipsoid method is used, $q = 2$). The complexity in (12) and the Hungarian method are $\mathcal{O}(MK)$ and $\mathcal{O}(N^3)$, respectively. Combining all, the total complexity of the proposed method is $\mathcal{O}((MK + N^3)K^q)$, which is polynomial.

If consider long-term fairness among the MSs, the weight of MS u at time t can be updated by $w_u^{(t)} = 1/T_u^{(t)}$, $\forall u \in \mathcal{U}$, where $T_u^{(t)}$ as the accumulated rate of MS u at time t . Note that we can let $w_u = 1$ for every MS for pure throughput maximization.

IV. SIMULATION RESULTS

We consider a cell with 2 km radius. The RSs are uniformly located on a circle centered at the BS and with radius of 1 km. The MSs are randomly but uniformly distributed in the outer circle as in Fig. 1. The path loss exponent is 4 and the standard deviation of log-normal shadowing is 5.8 dB. The small-scale fading is modeled by multi-path Rayleigh fading process. A total of 3000 independent channel realizations were used. Each channel realization is associated with a different set of node locations. We set $M = 4$, $K = 3$, and $N = 32$. All MS and the BS have the same maximum power constraints, so do all RSs. We set the BS and MS power to be 10 dB per-node and uniformly distributed among all subcarriers.

As the benchmarks, the performance of Equal Power Assignment (EPA) based resource allocation and Random Resource Allocation (RRA) schemes are also presented. Specifically, EPA lets p be uniformly distributed among all the subcarriers on each relay station and finds optimal ρ^* as in Section III-A proposed algorithm. In RRA, the power is uniformly distributed and the subcarrier pairs and relays are randomly assigned. The complexity of the EPA and RRA schemes are $\mathcal{O}(MK + N^3)$ and $\mathcal{O}(N)$, respectively, which are lower than that of the proposed algorithm.

Fig. 2 compares the average sum-rate achieved by different schemes. We first observe that the proposed dual-based algorithm approaches the upper bound (the optimal dual) closely. This verifies the effectiveness of the dual method at large number of subcarriers. One also observes that the proposed algorithm outperforms the two benchmarks by a significant margin. In particular, the proposed algorithm obtains more than 30% and 200% throughput improvements over the EPA and RRA schemes, respectively. This tremendous improvement demonstrates the superiority of our proposed algorithm.

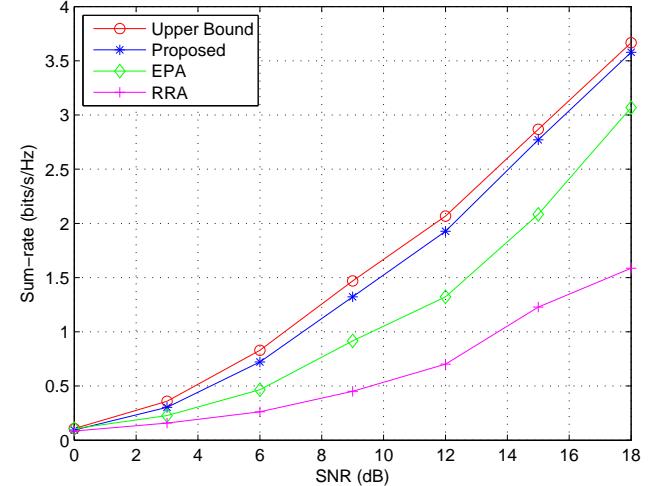


Fig. 2. Average sum-rate versus RS-power per-node.

V. CONCLUSION

In this work, we have studied the subcarrier-pairing based resource allocation in OFDMA-based two-way relay networks. By using the dual method, an efficient algorithm for joint optimization of subcarrier-pairing based relay-power allocation, relay selection, and subcarrier assignment was proposed. Simulation results show that the proposed algorithm can significantly improve the system performance compared with the conventional schemes.

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